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585 B. C. Eight times the 307-year cycle brings us to April 24, 1876. It does not appear that any eclipse will happen at that time.

Another eclipse that seems to be referred to by Xenophon happened, according to Prof. Airy, May 19, 556 B. C. Eight 307-year periods, minus one 57-year period, brings us to April 27, 1847. No eclipse happened at that time.

The eclipse of March 20, 1140 A. D., total at London, returned December 21, 1870, after two 307-year periods, plus two 57-year periods. The eclipse of August 21, 1560, returned February 22, 1868, after one 307-year period; and the eclipse of April 9, 1567, returns October 9, 1874, after one 307-year period.

Of instances of the 57-year period we may mention that the eclipse of July 14, 1748, returned in 1806, June 16th; and this last returned May 5, 1864. The great eclipse of June 24, 1778, returned in 1836, May 15th, as an annular eclipse, after one period.

So far as I have been able to compare these periods with eclipses whose dates are certain, they give the order of the eclipses.

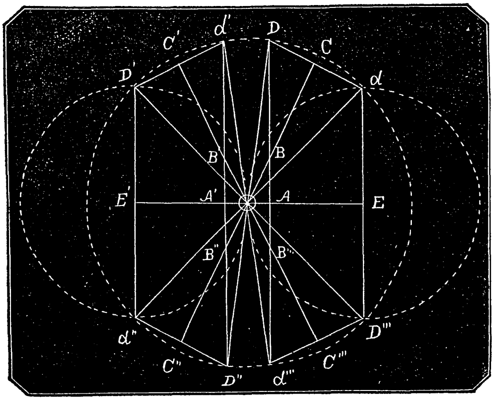
OPERATIONS ON IMAGINARY QUANTITIES CONSIDERED GEOMETRICALLY.

BY PROF. W. D. HENKLE, SALEM, OHIO.

In the view here taken of so-called imaginary quantities the $\sqrt{-1}$ is not considered as a quantity at all, but merely as a sign of an operation. First, let us consider the multiplication of binomial expressions. The following table of signs shows that there are sixteen cases:—

+ + + +	+ + + —	+ + — +	+ + — —
+ — + +	+ — + —	+ — — +	+ — — —
— + + +	— + + —	— + — +	— + — —
— — + +	— — + —	— — — +	— — — —

[1] $1+2\sqrt{-1}$ $\underline{3+\sqrt{-1}}$ $1+7\sqrt{-1}$	[2] $1+2\sqrt{-1}$ $\underline{3-\sqrt{-1}}$ $5+5\sqrt{-1}$	[3] $1+2\sqrt{-1}$ $\underline{-3+\sqrt{-1}}$ $-5-5\sqrt{-1}$	[4] $1+2\sqrt{-1}$ $\underline{-3-\sqrt{-1}}$ $-1-7\sqrt{-1}$
[5] $1-2\sqrt{-1}$ $\underline{3+\sqrt{-1}}$ $5-5\sqrt{-1}$	[6] $1-2\sqrt{-1}$ $\underline{3-\sqrt{-1}}$ $1-7\sqrt{-1}$	[7] $1-2\sqrt{-1}$ $\underline{-3+\sqrt{-1}}$ $-1+7\sqrt{-1}$	[8] $1-2\sqrt{-1}$ $\underline{-3-\sqrt{-1}}$ $-5+5\sqrt{-1}$
[9] $-1+2\sqrt{-1}$ $\underline{3+\sqrt{-1}}$ $-5+5\sqrt{-1}$	[10] $-1+2\sqrt{-1}$ $\underline{3-\sqrt{-1}}$ $-1+7\sqrt{-1}$	[11] $-1+2\sqrt{-1}$ $\underline{-3+\sqrt{-1}}$ $+1-7\sqrt{-1}$	[12] $-1+2\sqrt{-1}$ $\underline{-3-\sqrt{-1}}$ $+5-5\sqrt{-1}$
[13] $-1-2\sqrt{-1}$ $\underline{3+\sqrt{-1}}$ $-1-7\sqrt{-1}$	[14] $-1-2\sqrt{-1}$ $\underline{3-\sqrt{-1}}$ $-5-5\sqrt{-1}$	[15] $-1-2\sqrt{-1}$ $\underline{-3+\sqrt{-1}}$ $5+5\sqrt{-1}$	[16] $-1-2\sqrt{-1}$ $\underline{-3-\sqrt{-1}}$ $+1+7\sqrt{-1}$



In the first example, take $OA=1$ and $AB=2$ at right angles to OA, in consequence of the sign of operation $\sqrt{-1}$ with which 2 is affected; then OB represents $1+2\sqrt{-1}$ in magnitude and position.

Take $OC=3\,OB$; then OC represents 3 times $1+2\sqrt{-1}$. Draw $CD=OB$ at right angles to OC; then CD represents $\sqrt{-1}$ times $1+2\sqrt{-1}$ as added to 3 times $1+2\sqrt{-1}$. Hence OD represents the full product of $1+2\sqrt{-1}$ multiplied by $3+\sqrt{-1}$. It is easily proved that AB produced passes through D and that $AD=7$. Hence OD is also the construction of $1+7\sqrt{-1}$.

In the second example the $\sqrt{-1}$ of the multiplier being negative, $Cd=OB$ is drawn in the opposite direction from CD. This gives Od as the final product. Draw dE perpendicular OE. It is easily proved that OE and dE are each equal to 5. Hence Od is the construction of $5+5\sqrt{-1}$.

In the third example, the multiplier may be placed under the form $-(+3-\sqrt{-1})$; hence the result is the same as the result in the second example taken negatively. Od'' equals Od taken negatively. It is easily proved that OE' and $E'd''$ are each equal to 5. Hence Od'' is the construction of $-5-5\sqrt{-1}$.

It is plain that in the fourth example the multiplier differs from that in the first only in being negative. Hence the result is OD'' or OD taken negatively. It is also evident that OD'' is the construction of $-1-7\sqrt{-1}$.

In the fifth example, take $OA=1$ and $AB'''=2$, then OB''' is the construction of $1-2\sqrt{-1}$. Take $OC'''=3\ OB'''$and draw $C'''D'''$ at right angles to OC'''. The final product is OD''' This is also the construction of $5-5\sqrt{-1}$, since OE and ED''' can be proved to be each equal to 5.

In the sixth example $C'''d'''=OB'''$ is drawn on the other side of OC''' since $\sqrt{-1}$ is negative. The final product is Od''', which is also the construction of $1-7\sqrt{-1}$.

In the seventh example place $-3+\sqrt{-1}=-(+3-\sqrt{-1})$. The result must then be the same as that in the sixth taken negatively, that is Od' which is $-Od'''$ Od' is also the construction of $-1+7\sqrt{-1}$.

In the eighth example place $-3-\sqrt{-1}=-(+3+\sqrt{-1})$. Hence the result must be the negative of that in the fifth, that is $-OD'''$, or OD'. OD' is also the construction of $-5+5\sqrt{-1}$.

In the ninth example, take $OA'=-1$ and $A'B'=2$; then is OB' the construction of $-1+2\sqrt{-1}$. Take $OC'=3OB'$ and draw $C'D'=OB'$. The final product OD' is also the construction of $-5+5\sqrt{--1}$.

In the tenth example, draw $C'd'=OB'$. The final result Od' is also the construction of $-1+7\sqrt{-1}$.

In the eleventh example, place $-3+\sqrt{-1}=-(+3-\sqrt{-1})$. Hence the final product must be the negative of that in the tenth, or Od''', which is also the construction of $1-7\sqrt{-1}$.

In the twelvth example, place $-3-\sqrt{-1}=-(+3+\sqrt{-1})$. Hence the final product is the negative of that in the ninth, or OD''', which is also the construction of $5-5\sqrt{-1}$.

In the thirteenth example, take $OA'=-1$ and $A'B''=2$, then is OB'' the construction of $-1-2\sqrt{-1}$. Draw $OC''=3OB''$ end $C''D''=OB''$. Hence the final product is OD'', which is also the construction of $-1-7\sqrt{-1}$.

In the fourteenth example, draw $C''D''=OB''$. Hence the final product is Od'', which is also the construction of $-5-5\sqrt{-1}$.

In the fifteenth example, place $-3+\sqrt{-1}=-(+3-\sqrt{-1})$. Hence the final product is the negative of that in the fourteenth, or Od, which is also the construction of $5+5\sqrt{-1}$.

In the sixteenth place, $-3-\sqrt{-1}=-(+3+\sqrt{-1}$. Hence the product is the negative of that in the thirteenth, or OD, which is also the construction of $1+7\sqrt{-1}$.

The conventions adopted in these illustrations are the simplest that have occurred to me.

I may in a future article illustrate division from the same diagram, and addition and subtraction from other diagrams.

In the meantime let those who can give a geometrical illustration of an imaginary exponent.

EQUATIONS OF DIFFERENCES.

BY WALTER SIVERLY, OIL CITY, PA.

I am not aware that this subject has been discussed in any American work, and, so far as I know, in but few foreign ones. The only works that I have seen which treat upon this subject are "Hymer's Finite Differences" and "Demorgan's Differential and Integral Calculus."

The limits of this article will not allow me to introduce the discussions involved in the integration of the equations; but I will give, 1st, the formula resulting from the integration of an equation of the first degree, in the first order of differences, with an example of its application; and, 2nd, the formula resulting from the integration of an equation of the n^{th} degree, with an example also of its application. These formulæ can be used without further knowledge of the subject.

1. Let u_0, u_1 u_2, . . . u_{x-1}, u_x, u_{x+1}, &c., be any series of which u_x a function of x, is the general term; any relation between the terms in functions of x expressed by means of an equation, is called an equation of differences.

Let the relation be $u_{x+1} - Au_x = B$, A and B being constants. The integration of this equation gives $u_x = CA^x + \frac{B}{1-A}$, ...(1) C being an arbitrary constant determined as in ordinary integration.

Example.—One of two casks contains a gallons of wine and the other b gallons of water; c gallons are taken from the first and poured into the second cask, and then c gallons are taken from the second and poured into the first. Required the quantity of wine in the second cask after n such operations.

Let $u_n =$ the wine in the second cask after the nth operation, the first cask will then contain $a - u_n$ gallons of wine; at the next operation there will be taken out of the first cask

$$\frac{c}{a}(a - u_n)$$